the reduced frequency, are shown in Figs. 4a and 4b for $M_{S0} = 1.37$ and for two transducer positions $x_{T/C} = 0.78$ and 0.92.

The forcing function $F(n)$ is the contribution to $(\tilde{p}/q_\infty)^2$ in the nondimensional frequency band dn such that

$$(\tilde{p}/q_\infty)^2 = \int_{n=0}^{n=\infty} F(n)\mathrm{d}n$$

It is clear from these figures that the effect of PSBL is to reduce the levels at lower frequencies significantly (< 1 KHz) and to increase the levels slightly at higher frequencies. The solid surface model shows a peak in spectra at $n \approx 0.28$ at the shock position. This would correspond to a frequency parameter of $2\pi fc/U_\infty = 1.13$, which compares well with the other experimental values[6] measured at the shock position. The reduction in the pressure fluctuation levels at low frequencies will also reduce the buffeting associated with shock oscillations and shock-induced separation.

Conclusions

It may be concluded from these investigations that PSBL control in transonic flow can reduce pressure fluctuations in the region of shock/boundary-layer interaction and therefore suppress buffeting.

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Initial Instability in a Freejet Mixing Layer Measured by Laser Doppler Anemometry

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I. Introduction

MUCH attention has been focused in recent years on the instability waves that develop at the origin of free turbulent shear layer. Michalke[1] calculated the spatial growth of small perturbation in a shear layer with a finite thickness. He found that the most amplified instability frequency f_0 can be scaled with the initial momentum thickness Θ_0 and the jet exit velocity V_0. The Strouhal number based on this scaling was a constant:

$$St_0 = f_0 \frac{\Theta_0}{V_0} = 0.017$$

Many investigators confirmed[2-4] these calculations, the values varying from 0.01 to 0.018. The jet's initial momentum thickness varies with the exit velocity as $V_0^{-1/2}$. Hence, f_0 should be proportional to $V_0^{3/2}$. Contrary to this prediction, Gutmark and Ho[5] observed a stepwise variation of the initial instability frequency with the jet velocity, in several facilities. A possible cause for this discrepancy was interference by the hot-wire probe. Hussain and Zaman[6] have shown that when a probe is placed in the shear layer, it can trigger, by its presence, flow instabilities that have characteristics similar to those observed in the above-mentioned experiments. The purpose of the present study was to rule on this possibility by initializing a nonintrusive technique, namely, laser Doppler anemometry (LDA). This technique was used to measure the behavior of the initial instability of the jet, and the results were compared with those obtained by using the conventional hot-wire anemometer on the same system. In this study, velocity spectra were generated from the LDA velocity measurements with the aid of a spectral technique outlined in Sec. III. The results of the present work showed identical behavior of the initial instability frequency as reported by Gutmark and Ho.[5]

II. Facility and Instrumentation

The measurements reported in this paper were performed in an axisymmetric jet operating from a compressed air supply. A contraction section with a fifth-order polynomial profile and contraction ratio of 64 led from the stagnation chamber to a nozzle with a diameter D of 25.4 mm. Honeycombs and layers of fine screen were used to reduce the turbulence intensity. As a result, the turbulence level was less than 0.4% in the operating velocity range.

The laser Doppler anemometer, depicted in Fig. 1, is based on a dual-beam system operating in the forward scattering mode.[7] A continuous-wave (CW) argon-ion laser emitting 1.5 watts at 514.5 nm has been employed as the light source for the LDA. In order to obtain spectra with a few kHz bandwidth, one has to resort to artificial seeding of the jet flow. In this experiment the aerosol was generated with a Laskin nozzle and limited to small particles by a one-stage cascade impactor. The artificially generated aerosol was injected in the plenum of the jet.

LDA signal processing was carried out in the following way: The photomultiplier output was amplified, filtered, and processed by the Thermo Systmes Inc. burst counter. Using a minicomputer, these data were stored in files, each containing a string of Doppler frequencies and the respective time interval

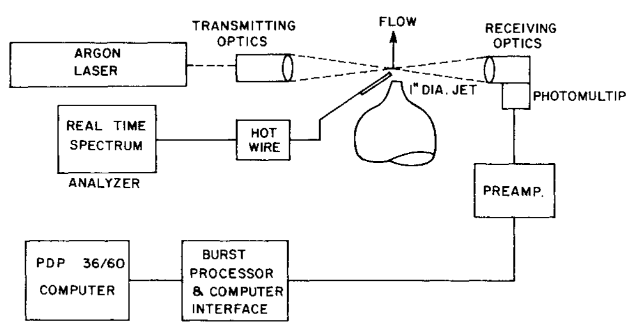

Fig. 1 Laser Doppler anemometer system.

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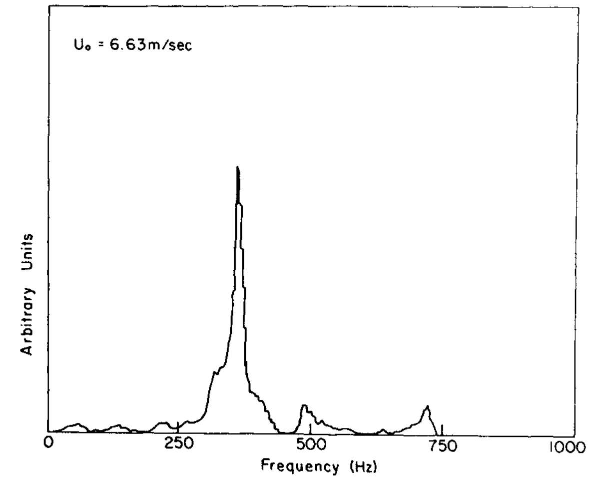

Fig. 2 Hot-wire spectra, computed by spectrum analyzer.

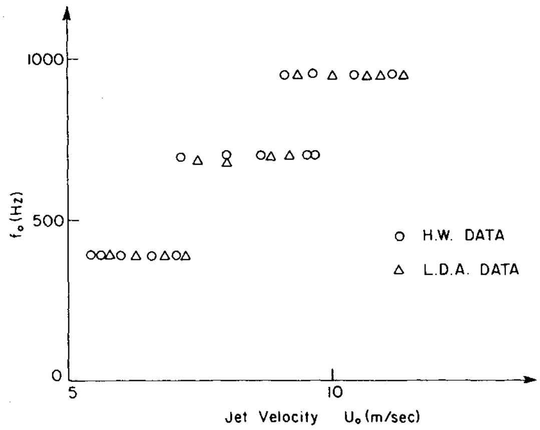

Fig. 3 Hot-wire spectra, computed by Doppler processing program.

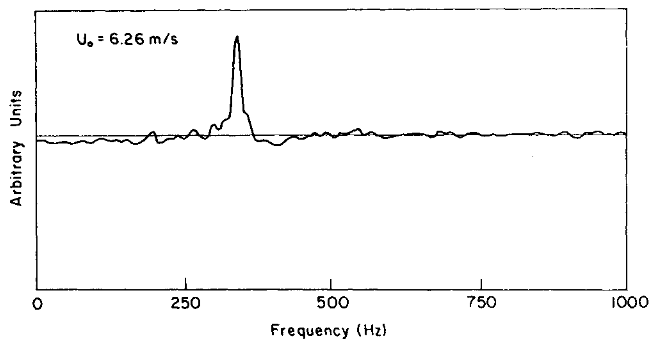

Fig. 4 Laser Doppler anemometer spectra.

between each two consecutive Doppler frequency bursts. These files were later retrieved for the spectral analysis as outlined in Sec. III.

Hot-wire data were concurrently taken with a constant-temperature DISA anemometer having a 20-kHz frequency response. A Saicor real-time analog spectrum analyzer served to generate spectra from the hot-wire signals.

III. Spectra Generation from LDA Signals

Many techniques have been developed for estimating spectra from equi-spaced samples. These techniques are inapplicable to data acquired in an irregular way. A specific example is the LDA operating in the burst mode, which produces a

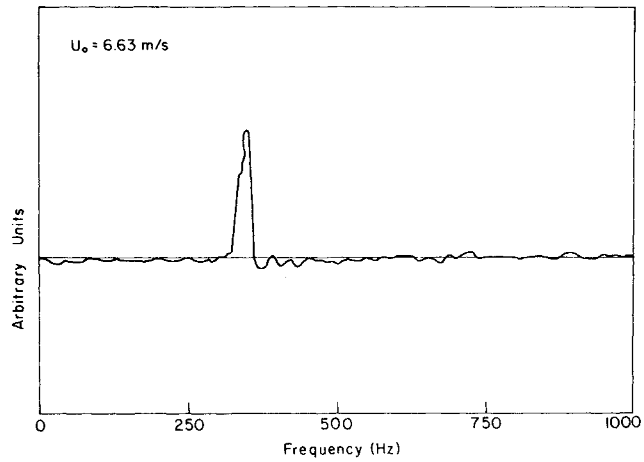

Fig. 5 Comparison of hot-wire and Laser Doppler anemometer results.

velocity trace whenever a particle is crossing the focusing volume and scattering radiation. In order to generate spectral estimates from randomly spaced LDA velocity traces, one has to make several assumptions, for example, that the time instants are Poisson distributed or that the analyzed blocks of data are limited in length but, on the other hand, long compared with the time scale of the signal. Detailed descriptions of the above technique are provided in several publications of one of the authors.[7-9]

IV. Experimental Results

Both LDA and hot-wire anemometers were used in determining the initial instability frequency. These measurements were made at identical measuring points located at a distance of $X/D = 0.3$ from the nozzle. At a radial distance inside the shear layer, where the local velocity is one-half the exit velocity, LDA data were obtained with and without a hot wire present in the shear layer.

A typical spectrum generated from the hot-wire output using the analog spectrum analyzer is presented in Fig. 2. One clearly observes the distinct frequency of 330 Hz present at this point in the flow under these conditions. When scaled with the initial momentum thickness and the exit velocity, the resulting Strouhal number was 0.013. This indicated that this frequency corresponded to the initial instability of the jet. Farther downstream, along the shear layer of the jet, the frequency was halved twice by the vortex merging process, yielding a preferred mode frequency of 84 Hz at the end of the jet potential core.

We used the following procedure to check out the program of generating spectra from LDA signals. The hot-wire output was superimposed on a 1-MHz sine wave and introduced through the burst processor into the minicomputer, generating LDA-like data files. These files were retrieved and processed as outlined, with a typical result shown in Fig. 3. Here again the distinct frequency spike of 330 Hz is present as in Fig. 2. In Fig. 4 we have spectra generated from the actual LDA signals. Both Figs. 3 and 4 are identical and with the same frequency for the spike. In all three cases, Figs. 2-4, the jet velocities were identical.

Having demonstrated the capability of the computer program to generate spectra from discrete LDA signals, we proceeded to measure the initial instability frequencies. Figure 5 shows LDA and hot-wire results of these frequencies measured over a range of velocities. One can observe the stepwise variations of the frequency with jet velocity and also the excellent agreement between intrusive (hot-wire) and nonintrusive (LDA) results.

In conclusion, our measurements confirm previous results obtained by using hot-wire probes, thus ruling out probe interference effects as a cause for the stepwise variation of initial instability frequency with jet exit velocity.

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Dynamic Buckling of Orthotropic Spherical Caps Supported by Elastic Media

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Introduction

THE buckling of isotropic shallow spherical shells has been the subject of extensive studies.[1-13] There are few investigations[14,15] dealing with the dynamic buckling of orthotropic shallow spherical shells. In these studies the effect of interaction with the supporting elastic media on the dynamic buckling of spherical caps has not been considered. The dynamic response of isotropic shallow spherical shells supported on a Winkler-Pasternak elastic foundation has been investigated recently.[16] The studies of the dynamic buckling of orthotropic shallow spherical shells with or without holes interacting with supporting elastic media are not available in the literature and are dealt with for the first time in this paper.

Von Kármán-Marguerre-type governing nonlinear partial differential equations for the orthotropic spherical caps on a Winkler-Pasternak[17] elastic foundation are employed; they are linearized using the quadratic extrapolation technique and solved iteratively using the Chebyshev series and Houbolt schemes for the space and time domains respectively.[14,16] The conditions of finiteness at the center for the full cap and free inner edge for the annular cap are assumed. Two criteria, namely, a sudden jump[3] in the average deflection response and the point of inflection[8] in the load vs maximum average deflection curve, are used to estimate the dynamic buckling loads. The influence of foundation stiffness, orthotropy, and annular ratio on the dynamic buckling load of immovably clamped and simply supported shallow spherical caps is investigated, and typical results are presented.

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Mathematical Analysis

The geometry of a shallow spherical shell on a Winkler-Pasternak elastic foundation is shown in Fig. 1, in which a and b are outer and inner radii, h is the thickness of the shell, K and G are foundation stiffnesses, and w and u are normal and meridional deflections, respectively. Considering the cylindrically orthotropic material whose elastic axis of orthotropy coincides with the axis of symmetry of the shallow spherical shell of constant curvature K^*, the governing partial differential equation of motion and compatibility condition in terms of dimensionless normal deflection $\bar{w}$ and stress function $\bar{\psi}$ for a thin shallow spherical shell supported by a Winkler-Pasternak elastic foundation and undergoing

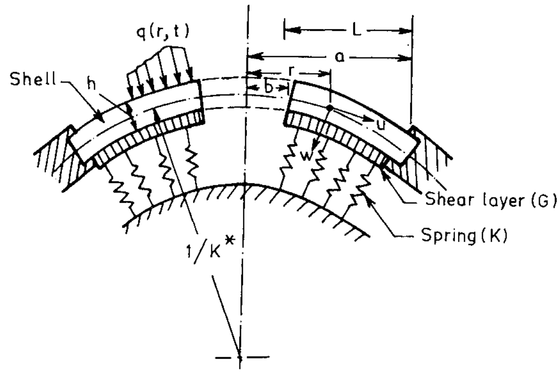

Fig. 1 Geometry of a shallow annular spherical cap on elastic foundation.

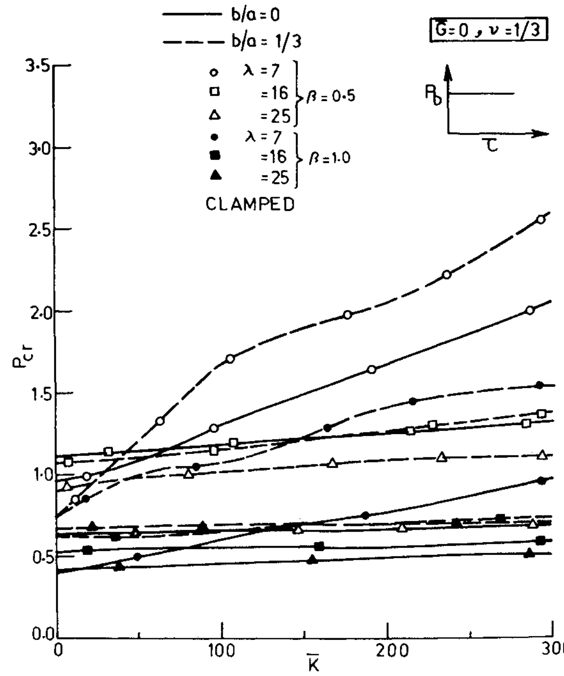

Fig. 2 Variation of dynamic buckling load P_{cr} of spherical caps with Winkler stiffness $\bar{K}$.